Discussion Problems 2

Problem One: Prime Numbers

A natural number $p \geq 2$ is called ***prime*** iff it has no positive divisors except 1 and itself. A natural number is called ***composite*** iff it is the product of two natural numbers m and n, where both m and n are greater than one. Every natural number greater than or equal to two is either prime or composite. Prove, by strong induction, that every natural number greater than or equal to two can be written as a product of prime numbers.

Problem Two: Picking Coins

Consider the following game for two players. Begin with a pile of n coins for some $n \geq 0$. The first player then takes between one and ten coins out of the pile, then the second player takes between one and ten coins out of the pile. This process repeats until some player has no coins to take; at this point, that player loses the game. Prove that if the pile begins with a multiple of eleven coins in it, the second player can always win.

Problem Three: Factorials! Multiplied together!

Prove by induction that for any $m, n \in \mathbb{N}$, we have $m!n! \leq (m + n)!$. Can you explain intuitively why this is?